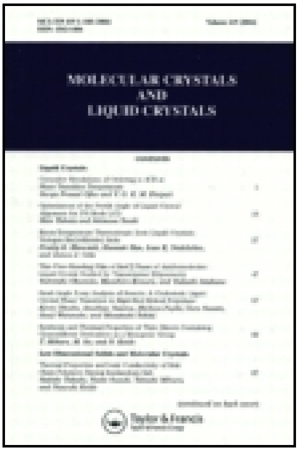

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Some New Results Concerning the Optical Properties of Anisotropic Stratified Media

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A general approach to the optics of anisotropic media, based on a new formalism is presented. It allows to straightforwardly obtain and generalize almost all previously known results in this research area.

INTRODUCTION

Most liquid-crystal devices involve the use of layered media. In particular, any liquid-crystal pixel can be considered as an anisotropic stratified medium whose macroscopic optical properties only depend on the distance z along the normal to the layers. A central problem of liquid-crystal optics is therefore the evaluation of transmittance and reflectance parameters for a plane electromagnetic wave incident on a stratified anisotropic medium. The same problem arises in treating the transmission of radio-waves in the ionosphere. In both cases, the Maxwell equations for electromagnetic-wave propagation can be cast in the following form[1,2]:

$$\frac{d|\psi\rangle}{dz} = i\frac{\omega}{c}D(z)|\psi\rangle \tag{1}$$

where $|\psi\rangle$ is the four-dimensional vector whose elements are the field components $E_x, H_y, E_y, -H_x$, and $D(z)$ is a 4×4 matrix whose elements are given in Appendix 1. It is noteworthy that Equation (1) has been independently derived in both research areas. Quite recently, a non-euclidean metric in the $|\psi\rangle$ vector space has been defined, such that the proper vectors $|\psi_1\rangle$, $|\psi_2\rangle$, $|\psi_3$, $|\psi_4\rangle$ of the matrix D for homogeneous non-absorbing media satisfy to the orthonormality relations

$$\langle\psi_i|\psi_j\rangle = \delta_{ij} \tag{2}$$

where $\langle\psi|$ is a suitably defined dual space of the $|\psi\rangle$ vector space.[3,4] For uniaxial media the proper vectors $|\psi_i\rangle$ correspond to the forward- and backward-propagating extraordinary and ordinary plane waves.

Moreover, the matrix D satisfies to the property[4]

$$\langle\psi|(D|\psi\rangle) = (\langle\psi|D)|\psi\rangle = \langle\psi|D|\psi\rangle \tag{3}$$

VARIATIONAL PRINCIPLE

It can be easily shown that the proper vectors $|\psi_i\rangle$ of a z-independent D matrix are such that the functional expression

$$d[|\psi\rangle] = \frac{\langle\psi|D|\psi\rangle}{\langle\psi|\psi\rangle} \tag{4}$$

is stationary. This result may be derived by making use of a standard variational procedure (see for instance Reference 5). The corresponding proper values d_i are proportional to the z-component of the wavevector $\bar{K}$. Stationarity of d means therefore stationarity of the phase difference between transmitted and incident wave. The interest for this property is evident, particularly in the light of the possible extension of Fermat's principle to anisotropic media. However, only some applications of the theory are outlined here.

These have been obtained by means of a standard perturbation-variation procedure: the matrix D is written as $D_0 + V$, where the proper values d_{0i} and vectors $|\psi_{0i}\rangle$ of D_0 are known, and V is treated as a perturbation.

Approximate expressions for the exact forward-propagating waves are obtained by considering only the subspace of the two unperturbed forward-propagating vectors $|\psi_{01}\rangle$, $|\psi_{02}\rangle$. These are obtained by setting $|\psi\rangle = f_i|\psi_{01}\rangle + f_2|\psi_{02}\rangle$ and solving the system

$$\begin{pmatrix} d_{01} - d + V_{11} & V_{12} \\ V_{21} & d_{02} - d + V_{22} \end{pmatrix} \begin{pmatrix} f_1 \\ f_2 \end{pmatrix} = 0 \tag{5}$$

which admits very simple analytic expressions, whereas the exact solution requires the diagonalization of a 4×4 matrix, usually involving heavy numerical procedures.

APPLICATIONS

In the authors' opinion, the described formalism is perhaps the most suitable one for a general approach to the optics of anisotropic media. In fact, it is particularly appropriate for simplified numerical computation; furthermore it gives a refined method, alternative to the standard one, for deriving exact or approximate analytic expressions in many branches of classical optics. Some of these are briefly reviewed here.

a) Plane Waves in Anisotropic Homogeneous Media

The first aim in classical optics of anisotropic media is to find out the plane waves with a given frequency and a given direction of the wavevector.

The subsequent step consists in finding the waves transmitted and reflected at the boundary between two different media. It is well known that for the latter problem exact analytic expressions are known only for optically inactive, uniaxial media.

All known expressions can be very easily found with the help of the present approach: in particular, simple analytic approximations are given in a straightforward way by means of Equation 5 and (a1) in some relevant cases, as for instance optical activity in uniaxial non absorbing media, induced biaxiality and conoscopy in biaxial crystals.

Optical activity (either natural or induced by a magnetic field) is related to the imaginary part ε'' of the dielectric tensor, which is in any case small enough to be treated as a perturbation. Similarly in all cases of electrically, magnetically or elastically induced biaxiality, the dielectric tensor variation is sufficiently small to be considered as a perturbation. On the other hand, in the case of conoscopy in biaxial crystals, the unperturbed waves correspond to the case of normal incidence, and the perturbation is the term depending on the tangential component of the wavevector $\vec{K}$.

b) Inhomogeneous Media

In many liquid-crystals devices the dielectric-tensor matrix and the matrix D are continuous functions of the coordinate z. The first step for the solution of the system of Equations (1) is the diagonalization of the matrix $D(z)$. Let $d_i(z)$ and $|\psi_i(z)\rangle$ be its proper values and vectors (either exact or approximated by means of the method previously described). By setting $|\psi(z)\rangle = \Sigma_{i=1}^{4} f_i|\psi_i(z)\rangle$, the system (1) can be written in the equivalent form[4]:

$$\frac{df_i}{dz} = i\frac{\omega}{c}\left(d_{0i} + \sum_{j\neq i} V_{ij}\right) f_i \tag{6}$$

where the quantities V_{ij} depend on the spatial derivatives of the dielectric tensor and are generally small, therefore playing the role of coupling terms.

The equation system (6) is the starting point for the approximations based on coupled-mode analysis or perturbation expansions. The simplest and perhaps most important approximation is the geometrical optics approximation (GOA), which is obtained[6] by neglecting all of the coupling terms V_{ij}, in Equation (6).

A more powerful approximation, generally referred to as the generalized geometrical-optics approximation (GGOA) is obtained by neglecting only the coupling terms between forward and backward-propagating waves.[7] The system (6) gives then:

$$\frac{d}{dz}\begin{pmatrix} f_1 \\ f_2 \end{pmatrix} = i\frac{\omega}{c}\begin{pmatrix} d_{01} & v_{12} \\ v_{12}^* & d_{02} \end{pmatrix}\begin{pmatrix} f_1 \\ f_2 \end{pmatrix} \tag{7}$$

where f_1 and f_2 are the components of the Jones vector,[8] representing the polarization state of the forward-propagating wave. The explicit expression of the quantities d_{0i} and V_{12} for locally uniaxial media is given in Appendix 2. The Equation (7) can be considered as the most general propagation equation for the Jones vector.

CONCLUSION

Same concepts of new type, recently introduced in liquid-crystal optics, have been briefly discussed. A variational principle and the propagation Equation (7) for the Jones vector are explicity given. However, the main interest of this work lies perhaps in that it outlines a general approach to the optics of anisotropic media, based on a new formalism, allowing one to obtain in a straightforward manner, generalized expressions equivalent to almost all previously known results in this research area.

APPENDIX 1

The most general form of the matrix D for non-magnetic materials is given by

$$\begin{bmatrix} -m_x \dfrac{\varepsilon_{zx}}{\varepsilon_{zz}} & 1 - \dfrac{m_x^2}{\varepsilon_{zz}} & -m_x \dfrac{\varepsilon_{zy}}{\varepsilon_{zz}} & -\dfrac{m_x m_y}{\varepsilon_{zz}} \\ \varepsilon_{xx} - m_y^2 - \dfrac{\varepsilon_{xz}\varepsilon_{zx}}{\varepsilon_{zz}} & m_x \dfrac{\varepsilon_{xz}}{\varepsilon_{zz}} & \varepsilon_{xy} + m_x m_y - \dfrac{\varepsilon_{xz}\varepsilon_{zy}}{\varepsilon_{zz}} & -m_y \dfrac{\varepsilon_{xz}}{\varepsilon_{zz}} \\ -m_y \dfrac{\varepsilon_{zx}}{\varepsilon_{zz}} & -m_x m_y \dfrac{1}{\varepsilon_{zz}} & -m_y \dfrac{\varepsilon_{zy}}{\varepsilon_{zz}} & 1 - \dfrac{m_y^2}{\varepsilon_{zz}} \\ \varepsilon_{yx} + m_x m_y - \dfrac{\varepsilon_{yz}\varepsilon_{zx}}{\varepsilon_{zz}} & -m_x \dfrac{\varepsilon_{yz}}{\varepsilon_{zz}} & \varepsilon_{yy} - m_x^2 - \dfrac{\varepsilon_{yz}\varepsilon_{zy}}{\varepsilon_{zz}} & -m_y \dfrac{\varepsilon_{yz}}{\varepsilon_{zz}} \end{bmatrix} \tag{a1}$$

where: $m_x = K_x/K_0$; $m_y = K_y/K_0$; $K_0 = \omega/c$; and the ε_{ij}'s, are the elements of the dielectric tensor. It should be reminded that in most applications it is always possible to set in which case the expression of D is considerably simplified.[1-4]

APPENDIX 2

The quantities d_{01}, d_{02}, d_{12} appearing in Equation (7) are given by:

$$d_{01} \equiv d_e = m_t \cos\varphi + (\varepsilon_t - m^2\varepsilon_t\varepsilon_f/\varepsilon_e\varepsilon_0)^{1/2}$$

$$d_{02} \equiv d_0 = (\varepsilon_0 - m^2)^{1/2}$$

$$v_{12} = i\frac{c}{\omega}c_e c_0\left\{\frac{m\sin\varphi}{\sin^2\vartheta}\left(1+\frac{d_e}{d_0}\right)\frac{d\vartheta}{dz} - \left[d_e + d_0 - m\cot\vartheta\cos\varphi\left(1+\frac{d_e}{d_0}\right)\right]\frac{d\varphi}{dz}\right\}$$

where

$$\frac{1}{c_e} = \sqrt{2}\left[d_e\left(1-\frac{m^2}{\varepsilon_0}\cos^2\varphi\right) - m\cos\varphi\cot\vartheta \left(1-\frac{m^2}{\varepsilon_0}+\frac{d_e^2}{\varepsilon_0}\right) + m^2\cot^2\vartheta\,\frac{d_e}{d_0}\right]^{1/2}$$

$$\frac{1}{c_0} = \sqrt{2}\left[d_0\cos^2\varphi + \frac{\varepsilon_0}{d_0}\sin^2\varphi - 2m\cos\varphi\cot\vartheta + \frac{m^2\cot^2\vartheta}{d_0}\right]^{1/2}$$

$$\frac{1}{\varepsilon_t} = \frac{\cos^2\vartheta}{\varepsilon_0} + \frac{\sin^2\vartheta}{\varepsilon_e}$$

$$\varepsilon_f = \varepsilon_0\sin^2\varphi + \varepsilon_t\cos^2\varphi$$

$$m = K_x/K_0;\ K_0 = \omega/c;\ K_y = 0$$

and ε_e, ε_0 are the extraordinary and ordinary principal values of the dielectric tensor, (ϑ, φ) the polar angles of the optical axis.

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